Optical lever detection in higher eigenmode dynamic atomic force microscopy

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The optical lever detection scheme is widely used in atomic force microscopy for the detection of the cantilever deflection. Laser spot size as well as adjustment of the laser along the cantilever determine the zeros of the transfer function of the signal path from the tip-sample forces to the optical readout. This can cause (almost) pole-zero cancellations which lead to a significantly reduced sensitivity in the detection of higher mode vibrations of the cantilever. Physically, the light lever detection integrates over the slope of the cantilever. However, the sign of the slope of higher flexural modes varies along the cantilever. Thus, integration can lead to a significantly decreased sensitivity to higher eigenmode vibrations. Illuminating only the area between the free end and the next zero crossing of the slope of the modal shape provides a good compromise between high and low frequency sensitivity. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1808058]

Higher eigenmodes of the vibrating cantilever have gained increasing interest in dynamic atomic force microscopy (AFM). Some experimental techniques based on higher mode oscillations are atomic force acoustic microscopy,[1,2] signal inverting dynamic AFM,[3] higher harmonic imaging,[4–6] and higher eigenmode dynamic force spectroscopy.[7] These AFM methods require a detection bandwidth covering several resonant frequencies of the cantilever. Apart from the design of the detection electronics the design layout of the elementary sensing system—cantilever and detection laser—is a crucial point in the realization of high-bandwidth AFM systems. It was shown that for contact mode AFM and force spectroscopy the optimum sensitivity is achieved for a laser spot of approximately the same size as the cantilever.[8–11]

This raises the questions how to design an instrument with optical lever readout for higher mode detection and how to adjust an existing instrument for a good performance. From a system theoretic point of view this requires to tailor the transfer function of the sensing system by balancing the static gain and the gain at higher resonances.

The microcantilever can be treated as a linear and time invariant multiple-degrees-of-freedom-system employing the state-space formalism.[12] The equations of motion are given by

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}u, \tag{1}$$

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}, \tag{2}$$

with the system matrix A, the input vector b, and the output matrix C. The state-vector $\boldsymbol{x}=(x_1,\dot{x}_1,\ldots,x_n,\dot{x}_n)$ contains the displacements and velocities for n modes. For simplicity, the quality factors of all flexural modes were set to $Q=100$. In the following, we assume that the tip is attached to the free end of the cantilever.

The components of the output vector $\boldsymbol{y}=[y_1,y_2]^T$, i.e., the tip displacement output y_1 and the photodiode signal output y_2, are linear combinations of the states as defined in the output matrix

$$\boldsymbol{C} = \begin{bmatrix} -2 & 0 & \cdots & 2 & 0 \\ c_1/n_0 & 0 & \cdots & c_{2n-1}/n_0 & 0 \end{bmatrix}. \tag{3}$$

The tip deflection output is represented by the first row. The output of the optical lever sensor is described by the second row in the matrix C. The coefficients c_i represent the effective coupling of the respective state (eigenmode) to the output signal. Scalar n_0 is a normalization coefficient. Equations (1) model a dynamic system with the tip-sample forces as system input and the tip deflection and the photodiode signal as system outputs. In the following we will discuss the transfer function from the tip-sample force input to the optical lever output.

In order to determine the coefficients c_i the reflection of the laser light on the curved cantilever has to be considered. The geometrical shape of a vibrating cantilever beam can be approximately described[13] by the eigenvectors of

$$EI\frac{\partial^4 z(x,t)}{\partial x^4} + m\frac{\partial^2 z(x,t)}{\partial t^2} = 0, \tag{4}$$

with the normalized coordinate $x \in [0,1]$, the time t, the flexural stiffness EI, and the mass per unit length m, assuming boundary conditions for a clamped end at $x=0$ and free end at $x=1$. For a free cantilever the eigenvector (modal shape) of the ith mode (eigenvalue k_i) is given by

$$\varphi_i(x) = \cos k_i x - \cosh k_i x - \frac{\cos k_i + \cosh k_i}{\sin k_i + \sinh k_i} \times (\sin k_i x - \sinh k_i x). \tag{5}$$

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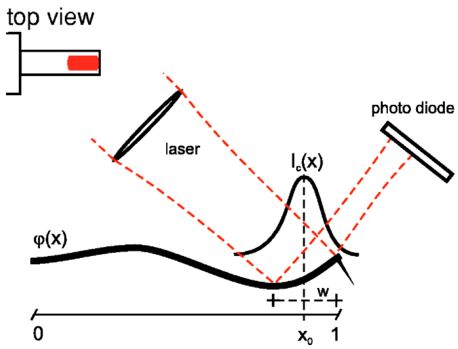

FIG. 1. (Color online) Scheme of the light lever detection in atomic force microscopy. The inset (top left) illustrates the illumination by the detection laser with a rectangular cross section.

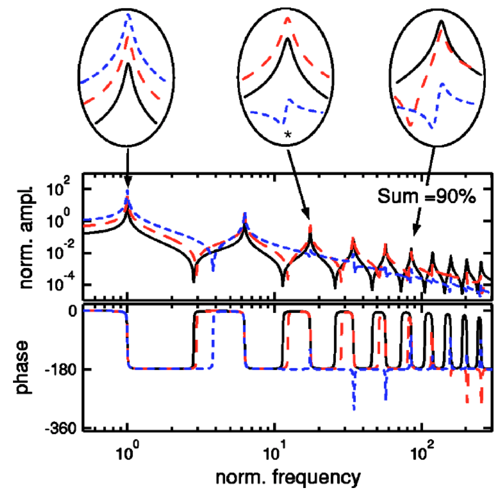

FIG. 2. (Color online). Bode plot of the transfer functions $G^w(\omega)$ for $w=0.9$ (blue, short dash), $w=0.3$ (red, dash), and $w=0.1$ (black, solid). The sum signal is 90% for all curves. The insets (not to scale) highlight details of the transfer functions where $\|G^{0.1}(\omega)\|<\|G^{0.3}(\omega)\|<\|G^{0.9}(\omega)\|$, $\|G^{0.9}(\omega)\|<\|G^{0.1}(\omega)\|<\|G^{0.3}(\omega)\|$, and $\|G^{0.9}(\omega)\|<\|G^{0.3}(\omega)\|<\|G^{0.1}(\omega)\|$. Note that for $w=0.9$ the third eigenmode is nearly undetectable due to an almost pole-zero cancellation (asterisk).

For the detection laser we assume a normalized Gaussian beam profile with the irradiance[11]

$$I_C(x) = I_0 \exp\left[-8\frac{(x-x_0)^2}{w_2}\right] \tag{6}$$

with $I_0=\sqrt{8/\pi}P_0/w$. Parameters are the total light power, P_0, the $1/e^2$ width along the cantilever axis w, and the position of the spot center on the cantilever beam x_0, as illustrated in Fig. 1. The spot size w is normalized to the cantilever length. The corresponding real scalar field is

$$E_C(x,x_0) = \sqrt{I_0}\exp\left[-\frac{[2(x-x_0)]^2}{w^2}\right]. \tag{7}$$

In the limit of small deflections ($A\varphi(x)\ll\lambda, A$ is the oscillation amplitude) the modal output coupling factor is $c_i(x_0)=S_i^-/A$, where the difference signal, S_i^-, is given by Eq. (10) in Ref. 10. This leads to

$$c_i(x_0) = \beta\int_0^1\int_0^1 dx\,dx' E_c(x,x_0)E_c(x',x_0)\frac{\varphi_i(x)-\varphi_i(x')}{x-x'}. \tag{8}$$

The parameter β is a coefficient that corrects for loss in the optics and includes the laser wave length. Equation (8) implies that the components of the output matrix C depend on both parameters, laser spot adjustment x_0, and laser spot size w. The detection optics is assumed to be constant.

For contact mode and force spectroscopy—both are static applications—it was shown[11] that the maximum detection sensitivity can be obtained with a spot diameter of about $w=0.9$. However, it is not immediately clear which spot size w provides optimum sensitivity with respect to higher eigenmodes. To compare the detection performance of different spot sizes the following laser alignment procedure is assumed: (i) adjust the laser to obtain the maximum sum signal (ii) move the laser to the free end of the cantilever until the sum signal is reduced to 90% of its maximum value. Obviously, this alignment procedure does not provide optimum sensitivity since 10% of the total laser power are lost. However, such a procedure allows the experimentalist to adjust the laser spot to the free end without the need of a high resolution viewing optics.

The corresponding transfer functions are illustrated in Fig. 2. The transfer function $G^w(\omega)$, was calculated for spot diameters of $w=0.1, w=0.3$, and $w=0.9$ and normalized to the maximum static sensitivity for a spot diameter of $w=0.9$ ($x_0=0.59$, sum signal 99.9%). This leads to a normalization with respect to the optimum static sensitivity. The static gains were $\|G^{0.1}(0)\|=0.13$, $\|G^{0.3}(0)\|=0.38$, and $\|G^{0.9}(0)\|=0.89$. This is in line with observation that for low frequency applications a $1/e^2$-spot diameter of approximately the length of the cantilever provides the optimum sensitivity.[8,9,11] However, the amplitude response of $G^{0.3}(\omega)$ outperforms that of $G^{0.9}(\omega)$ for the eigenmodes $n\geq 3$ (arrow) and $G^{0.1}(\omega)$ outperforms $G^{0.3}(\omega)$ for $n\geq 6$ (arrow). This can be understood considering the integrating behavior of the light lever detection that measures the integrated slope of the illuminated region. The signal contributions from regions with negative and positive slope cancel which leads to a reduced signal. More precisely, the root of $\phi_3'(x)=0$ is $x\approx 0.7[\phi_6'(0.9)=0]$. The $1/e^2$ width of the laser spot is then of the same size as compared to the region of the modal shape with the same sign of slope (Fig. 1). Additionally, in $G^{0.9}(\omega)$ the third eigenmode is nearly canceled out. This illustrates that higher eigenmodes can even be undetectable if the laser spot is not adjusted appropriately.

Thus, the laser should only illuminate the region between the free end of the cantilever $x=1$ and the next zero crossing of the slope x_i with $\phi_i'(x_i)=0$ of the highest flexural mode i to be detected. This provides a good compromise between low and high frequency sensitivity. A further increase of the laser spot size would increase the static gain but would also reduce the gain in the high frequency range. On the other side, a reduction of the spot size would reduce the gain in the entire frequency band of interest.

Another question is the influence of laser adjustment on the transfer function. Figure 3 shows the amplitude response (solid) for $w=0.9$ and $w=0.1$ for a laser beam adjustment at the free end ($x_0=1.0$) and in the middle ($x_0=0.5$) of the cantilever. The dotted lines indicate the idealized amplitude re-

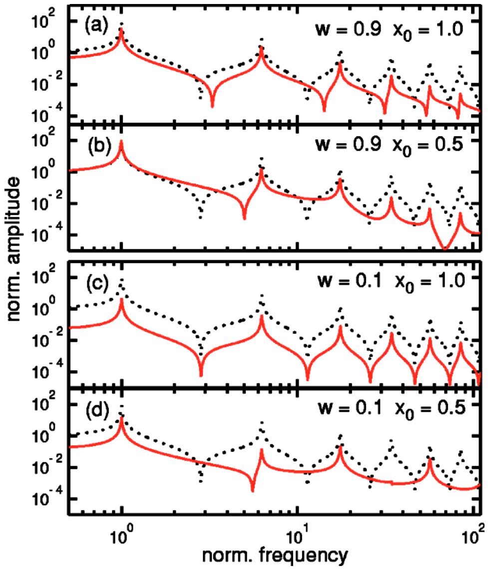

FIG. 3. (Color online) Comparison of the amplitude response (solid) for a detection laser spot size of (a),(b) $w=0.9$ and (c),(d) $w=0.1$. Dotted lines indicate the amplitude response of an idealized slope detector with an infinitely small spot size. (a) Sacrificing half of the laser power the transfer function still reproduces the higher resonances. (b) Moving the laser spot to the middle increases the static gain but reduces the sensitivity for higher modes and shifts zeros to higher frequencies. (c) With a smaller laser spot the zeros of the idealized transfer function are well approximated at the cost of gain. (d) Several transmission zeros are shifted outside of the frequency range of the plot. The fourth mode is undetectable.

sponse assuming a perfect slope detection at the free end of the cantilever neglecting the finite spot size[12] for comparison. From Fig. 3 it is obvious that the resonant frequencies (poles) do not depend on the adjustment of the laser beam or the laser spot size. However, the frequencies of the transmission minima (zeros) critically depend on the laser beam adjustment. This can be expected since the poles of a transfer function do not depend on the sensor location whereas the zeros do.[14] The zeros are moved to higher frequencies or become non-minimum phase by shifting the laser spot from the free end toward the middle of the cantilever [compare Fig. 3(a) with 3(b) and Fig. 3(c) with 3(d)]. In Fig. 3(d) the second mode is nearly canceled and the forth mode is completely suppressed. This implies that the transfer function strongly depends on the position of the laser spot along the cantilever axis: Any adjustment of the laser positioning significantly manipulates the transfer function of the force sensor. As discussed before, a variation of the laser spot size also manipulates the transmission zeros [compare Fig. 3(a) with 3(c), and 3(b) with 3(d)].

Usually, the experimentalist cannot change the laser spot size in a commercial AFM system. But even if the $1/e^2$ width of the spot exceeds the optimum size there is still room for improvement. Figures 3(a) and 3(b) illustrate a guideline that can be followed in that case. In low frequency applications such as contact mode and standard tapping mode the sum signal should be maximized [Fig. 3(b)]. In contrast, for high frequency experiments the laser spot should be moved to the end of the cantilever accepting a loss of the sum signal in order to increase the high frequency sensitivity [Fig. 3(a)].

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